MATHEMATICS

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AIM POINTS
MATHEMATICS
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AREA OF BOUNDED REGIONS

& Their Properties

CONTENTS

THINGS TO REMEMBER

1. Let f(x) be a continuous function defined on [a, b]. Then, the area bounded by the curve y = f(x), the x-axis and the ordinates x = a and x = b is given by $\int_a^b f(x)\,dx$ or, $\int_a^b y\,dx$
2. If the curve y = f(x) lies below x-axis, then the area bounded by the curve y = f(x), the x-axis and the ordinates x = a and x = b is negative. So, area is given by $\left|\int_a^b y\,dx\right|$
3. The area bounded by the curve x = f(y), the y-axis and the abscissae y = c and y = d is given by $\int_c^d f(y)\,dy$ or, $\int_c^d x\,dy$

EXERCISE-1

1. Let f(x) be a continuous function defined on [a, b]. Then, the area bounded by the curve y = f(x), the x-axis and the ordinates x = a and x = b is given by $\int_a^b f(x)\,dx$ or, $\int_a^b y\,dx$
2. If the curve y = f(x) lies below x-axis, then the area bounded by the curve y = f(x), the x-axis and the ordinates x = a and x = b is negative. So, area is given by $\left|\int_a^b y\,dx\right|$
3. The area bounded by the curve x = f(y), the y-axis and the abscissae y = c and y = d is given by $\int_c^d f(y)\,dy$ or, $\int_c^d x\,dy$
4. Find the area of the region included between the parabola $y = \frac{3x^2}{4}$ and the line 3x – 2y + 12 = 0.
5. Find the are bounded by the curve $x^2 = 4y$ and the straight line x = 4y – 2.
6. Find the area of the region enclosed by the parabola $y^2 = 4ax$ and the chord y = mx.
7. Find the area of the region included between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$, where a > 0.
8. Find the area of the region $\{(x, y) : x^2 \le y \le x$.
9. Find the area of the region $[(x, y) : x^2 \le y \le |x|\}$.
10. Find the area of the region $[(x, y) : 0 \le y \le x^2 + 1, 0 \le y \le x + 1, 0 \le x \le 2]$.
11. Find the area bounded by the curve $y^2 = 4ax$ and the lines y = 2a and y-axis.
12. Find the area bounded by the curve $y^2 = 4a^2 (x - 1)$ and the lines x = 1 and y = 4a.
13. Find the area of the region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

14. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$ and the straight line $\frac{x}{a}+\frac{y}{b} = 1$.
15. Find the area of the region $\{(x, y) : x^2 + y^2 \le 1 \le x + y\}$.
16. Find the area of the region $\{(x, y) : y^2 \le 4x, 4x^2 + 4y^2 \le 9\}$.

Or,

Find the area of the circle $4x^2 + 4y^2 = 9$ which is interior to the parabola $x^2 = 4y$.

17. Find the area of the region $\{(x, y) : x^2 + y^2 \le 2ax, y^2 \ge ax, x \ge 0, y \ge 0\}$.
18. Find the area of the region enclosed between the two circles $x^2 + y^2 = 1$ and $(x - 1)^2 + y^2 = 1$.
19. Using integration, find the area of the region bounded by the line $2y = -x + 8$, x-axis and the lines $x = 2$ and $x = 4$.
20. Using integration, find the area of the triangle ABC whose vertices have coordinates A (2, 5), B (4, 7) and C (6, 2).
21. Complete the area bounded by the line $x + 2y = 2$, $y - x = 1$ and $2x + y = 7$.
22. Using integration, find the area of the region bounded by the following curves, after making a rough sketch : $y = 1 + | x + 1 |$, $x = -3$, $x = 3$, $y = 0$.
23. Sketch the graph $y = | x + 1 |$. Evaluate $\int_{-3}^{1} |x+1|\,dx$. What does this value represent on the graph ?
24. Draw a rough sketch of the curves $y = \sin x$ and $y = \cos x$ as x varies from 0 to $\frac{\pi}{2}$ and find the area of the region enclosed by term and x-axis.
25. Find the area bounded by the curvey = sin x between $x = 0$ and $x = 2\pi$.
26. Sketch the region common to the circle $x^2 + y^2 = 16$ and the parabola $x^2 = 6y$. Also, find the area of the region using integration.

Or,

Using integration, find the area of the region $\{(x, y) : x^2 + y^2 \le 16, x^2 \le 6y\}$.

27. Prove that the curves $y^2 = 4x$ and $x^2 = 4y$ divide the area of the square bounded by $x = 0$, $y = 0$, $x = 4$ and $y = 4$ into three equal parts.
28. If the area enclosed between the curves $y = ax^2$ and $x = ay^2$ $(a > 0)$ is 1 square unit, then find the value of a.
29. Find the area of the region bounded by the line $y = 3x + 2$, the x-axis and the gradinates $x = -1$ ad $x = 1$.
30. Find the area bounded by the curve $y = x |x|$, x-axis and the ordinates $x = -3$ and $x = 3$.
31. Make a rough sketch of the graph of the function $y = 4 - x^2$, $0 \le x \le 2$ and determine the area enclosed by the curve, the x-axis and the line $x = 0$ and $x = 2$.
32. Sketch the graph of $y = \sqrt{x+1}$ in [0, 4] and determine the area of the region enclosed by the curve,

the x-axis and the lines $x = 0$, $x = 4$.

33. Draw a rough sketch of the graph of the curve $\frac{x^2}{4}+\frac{y^2}{9}=1$ and evaluate the area of the region under the curve and above the x-axis.

34. Find the area of the region $\left\{(x,y):\frac{x^2}{a^2}+\frac{y^2}{b^2}\le 1\le\frac{x}{a}+\frac{y}{b}\right\}$.

35. Using integration, find the area of the region bounded by the triangle whose vertices area (2, 1), (3, 4) and (5, 2).

36. Using integration, find the area of the triangular region, the equations of whose sides area $y = 2x + 1$, $y = 3x + 1$ and $x = 4$.

37. Find the area of the region between the circles $x^2 + y^2 = 4$ and $(x - 2)^2 + y^2 = 4$.

38. Sketch the graph $y = |x - 5|$. Evaluate $\int_0^1 |x-5|dx$. What does this value of the integral represent on the graph.

39. Sketch the graph $y = |x + 3|$. Evaluate $\int_{-6}^{0} |x+3|dx$. What does this integral represent on the graph ?

40. Draw a rough sketch of the curve $y = \frac{x}{\pi} + 2\sin^2 x$ and find the area between the x-axis, the curve and the ordinates $x = 0$ and $x = \pi$.

41. Find the area bounded by the curve $y = \cos x$, x-axis and the ordinates $x = 0$ and $x = 2\pi$.

42. Make a rough sketch of each of the following curves and determine the area of the region bounded by the curve and the axes :

(i) $y = \sin 2x$, $0 \le x \le \frac{\pi}{2}$

(ii) $y = \cos^2 x$, $0 \le x \le \frac{\pi}{2}$

(iii) $y = \cos 2x$, $0 \le x \le \frac{\pi}{4}$

(iv) $y = \cos 3x$, $0 \le x \le \frac{\pi}{6}$

(v) $y = \sin^2\frac{x}{2}$, $0 \le x \le \frac{\pi}{2}$

(vi) $y = \sin^2 x$, $0 \le x \le \frac{\pi}{4}$

(vii) $y = \sin^2 x$, $0 \le x \le \frac{\pi}{2}$

43. Find the area common to the circle $x^2 + y^2 + 16a^2$ and the parabola $y^2 = 6ax$.

44. Find the area, lying above x-axis and included between the circle $x^2 + y^2 = 8x$ and the parabola $y^2 = 4x$.

45. Prove that the area common to the two parabolas $y = 2x^2$ and $y = x^2 + 4$ is $\frac{32}{3}$ sq. units.

46. Compute the area of the figure bounded by the straight lines $x = 0$, $x = 2$ and the curves $y = 2^x$, $y = 2x - x^2$.

47. Find the area bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the ordinates $x = ae$ and $x = 0$, where $b^2 = a^2(1 - e^2)$ and $e < 1$.

48. Find the area of the region enclosed by the parabola $x^2 = y$ and the line $y = x + 2$.

49. Make a sketch of the region given below and find its area using integration.
$\{(x, y) : 0 \le y \le x^2 + 3; 0 \le y \le 2x + 3; 0 \le x \le 3\}$

50. Using integration, find the area of the region bounded by the line $y - 1 = x$, the x-axis and the ordinates $x = -2$ and $x = 3$.

51. Find the area bounded by the lines $y = 4x + 5$, $y = 5 - x$ and $4y = x + 5$.

52. Find the area of the region enclosed between the two curves $x^2 + y^2 = 9$ and $(x - 3)^2 + y^2 = 9$.

53. Using integration, find the area of the region enclosed between the circles $x^2 + y^2 = 4$ and $(x - 2)^2 + y^2 = 4$.

54. Using integration, find the area of the following region : $\left\{(x,y):\frac{x^2}{9}+\frac{y^2}{4}\le 1\le \frac{x}{3}+\frac{y}{2}\right\}$

EXERCISE-2

1. If the area above the x-axis, bounded by the curves $y = 2^{kx}$ and $x = 0$, and $x = 2$ is $\frac{3}{\log_e 2}$, then the value of k is
(a) 1/2 (b) 1 (c) –1 (d) 2

2. The area included between the parabolas $y^2 = 4x$ and $x^2 = 4y$ is (in square units)
(a) 4/3 (b) 1/3 (c) 16/3 (d) 8/3

3. The area bounded by the curve $y = \log_e x$ and x-axis and the straight line $x = e$ is
(a) e sq. units (b) 1 sq. units (c) $1 - \frac{1}{e}$ sq. units (d) $1 + \frac{1}{e}$ sq. units

4. The area bounded by $y = 2 - x^2$ and $x + y = 0$ is
(a) $\frac{7}{2}$ sq. units (b) $\frac{9}{2}$ sq. units (c) 9 sq. units (d) none of these

5. The area bounded by the parabola $x = 4 - y^2$ and y-axis, in square units, is
(a) $\frac{3}{32}$ (b) $\frac{32}{3}$ (c) $\frac{33}{2}$ (d) $\frac{16}{3}$

6. If A_n be the area bounded by the curve $y = (\tan x)^n$ and the lines $x = 0$, $y = 0$ and $x = \pi/4$, then for $x > 2$.
(a) $A_n + A_{n-2} = \frac{1}{n-1}$ (b) $A_n + A_{n-2} < \frac{1}{n-1}$ (c) $A_n - A_{n-2} = \frac{1}{n-1}$ (d) none of these

7. The area enclosed between the curves $y = \log_e (x + e)$, $x = \log_e\left(\frac{1}{y}\right)$ and the x-axis is

(a) 2 (b) 1 (c) 4 (d) none of these

8. The area bounded by the curves $y = \sin x$ between the ordinates $x = 0$, $x = \pi$ and the x-axis is
(a) 2 sq. units (b) 4 sq. units (c) 3 sq. units (d) 1 sq. units

9. The area bounded by the parabola $y^2 = 4ax$ and $x^{2e} = 4ay$ is

(a) $\frac{8a^3}{3}$ (b) $\frac{16a^2}{3}$ (c) $\frac{32a^2}{3}$ (d) $\frac{64a^2}{3}$

10. The area bounded by the parabola $y^2 = 4ax$, latusrecturm and x-axis is

(a) 0 (b) $\frac{4}{3}a^2$ (c) $\frac{2}{3}a^2$ (d) $\frac{a^2}{3}$

11. The area of the region $\{(x, y) : x^2 + y^2 \leq 1 \leq x + y\}$ is

(a) $\frac{\pi}{5}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{2} - \frac{1}{2}$ (d) $\frac{\pi^2}{2}$

12. The area of the region bounded by parabola $y = x^2 + 1$ and the straight line $x + y = 2$ is given by

(a) $\frac{45}{7}$ (b) $\frac{25}{4}$ (c) $\frac{\pi}{18}$ (d) $\frac{9}{2}$

13. The area between x-axis and curve $y = \cos x$ when $0 \leq x \leq 2\pi$ is
(a) 0 (b) 2 (c) 3 (d) 4

14. The area bounded by the curve $y = 4x - x^2$ and the x-axis is

(a) $\frac{30}{7}$ sq. units (b) $\frac{31}{7}$ sq. units (c) $\frac{32}{3}$ sq. units (d) $\frac{34}{3}$ sq. units

15. The area of the region (in square units) bounded by the curve $x^2 = 4y$, line $x = 2$ and x-axis is
(a) 1 (b) 2/3 (c) 4/3 (d) 8/3

16. The area bounded by the curve $y^2 = 8x$ and $x^2 = 8y$ is

(a) $\frac{16}{3}$ sq. units (b) $\frac{3}{16}$ sq. units (c) $\frac{14}{3}$ sq. units (d) $\frac{3}{14}$ sq. units

17. The area bounded by the y-axis, $y = \cos x$ and $y = \sin x$ when $0 \leq x \leq \frac{\pi}{2}$ is

(a) $2(\sqrt{2} - 1)$ (b) $\sqrt{2} - 1$ (c) $\sqrt{2} + 1$ (d) $\sqrt{2}$